A two-objective optimization of ship itineraries for a cruise company

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ABSTRACT
This paper deals with the problem of cruise itinerary planning which plays a central role in worldwide cruise ship tourism. In particular, the Day–by–day Cruise Itinerary Optimization (DCIO) problem is considered. Assuming that a cruise has been planned in terms of homeports and journey duration, the DCIO problem consist in determining the daily schedule of each itinerary so that some Key Performance Indicators are optimized. The schedule of an itinerary, i.e. the sequence of visited ports, the arrival and departure time at each port, greatly affect cruise operative costs and attractiveness. We propose a Mixed Integer Linear Programming (MILP) formulation of the problem with the objective of minimizing the itinerary cost due to fuel and port costs, while maximizing an itinerary attractiveness index which is strongly related to the ports visited as well as to the overall schedule of the itinerary. Therefore the problem turns out to be a bi–objective optimization problem. We solve exactly the MILP problem using a commercial solver.
We consider the day–by–day itineraries of a major luxury cruise company in many geographical areas all over the world. Here we report, as illustrative examples, the results obtained on some of these real instances.

KEYWORDS
Cruise shipping; Cruise itinerary optimization; Itinerary cost and attractiveness; Mixed Integer Linear Programming; Multiobjective optimization.

1. Introduction and literature review

In the last two decades, cruise shipping represented one of the most growing sectors of the shipping industry. According to Cruise Lines International Association (CLIA), the world's largest cruise industry trade organization (see (CLIA, 2021) for the latest report), since 1990 this sector has grown at an average annual passenger rate of 7.4% (see also (Cruise Market Watch, 2020)). Moreover, in the annual CLIA Global Market Report and State of Cruise Industry Outlook, the significant role played in international tourism by cruises is clearly evidenced: in 2019, the global cruise industry involved nearly 30 million passengers, creating jobs for 1.8 million people and contributing over $154 billion to the world economy. Of course the occurrence of the COVID-19 pandemic has dramatically hit the cruise industry, and all expectations for 2020 had to be drastically reconsidered (see e.g. (CLIA, 2021) and (Notteboom, Pallis, & Rodrigue, 2021, Chapter 1.5)). However the sector relies on a sound financial

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resilience, able to support the hope for better times in the next future A revival of the sector is also expected in the medium term thanks to the adoption of new procedures with enhanced protocols related to passenger health and to the proposal of new attractive itineraries: according to *Seatrade Cruise News*[1], 27 new oceangoing cruise ships will be launched during 2021.

Cruise itinerary planning plays a fundamental role in the strategic decisions of a cruise company. Indeed, itineraries are announced in advance and they should attract booking as much as possible. Actually, itinerary planning is the last step in the decision making process of any cruise company which is usually characterized by the following three levels: *i) the cruise fleet planning*, which is the highest level consisting in locating the ships in particular geographic areas in suited season windows so as to ensure the best weather conditions; *ii) the ship deployment*, i.e. to decide which cruises must be planned in the chosen areas in terms of embarkation port and disembarkation port (named turnaround ports or homeports) and the cruise duration; *iii) the day–by–day itinerary planning*, i.e., given the turnaround ports, to determine the sequence of intermediate ports (named ports of call or transit ports) to be visited by a ship and the arrival and departure time at each port. Note that there exist turnaround ports which can also act as transit ports.

In the previous paper (Di Pillo, Fabiano, Lucidi, & Roma, 2020), we have considered the second issue, namely the Cruise Itinerary Optimal Scheduling problem, aiming at determining a scheduling of cruises with the objective to maximize the revenue provided by a given ship placed in a specified maritime area, in a selected season window. Here we focus on the third issue, aiming at determining an optimal sequence of ports for the daily itinerary planning, minimizing the overall costs while maximizing customer satisfaction and taking into account several constraints on the itinerary design. In particular, we consider the cruise luxury market and this implies additional constraints usually not required by the cruise mass market. In fact, this latter usually offers to customers itineraries which are loops starting and ending at the same turnaround port and they are often repeated on week basis. Conversely, since customers of the cruise luxury market are usually returning customers, new itineraries, different from those already tried out, must be frequently proposed (see e.g. (Barron & Greenwood, 2006)) and in many cases they are one–way itineraries (starting and ending turnaround ports do not coincide). Of course, in the ship deployment phase, the turnaround ports are usually chosen close to international airports and on the basis of infrastructure and services of the ports. Transit ports in the day–by–day itinerary planning phase are selected on the basis of a number of factors (see e.g. (Sigala, 2017)) that go beyond port geographic location and availability of good facilities. Itinerary design is a critical issue for the success of a cruise since it strongly affects customers' choice and hence it has a great impact on the occupancy rate of a cruise ship (see (Jeon, Duru, & Yeo, 2019; Lee & Ramdeen, 2013)). Different sequences of the visited ports usually result in different logistic organization, possibly improving customer satisfaction. Moreover it is very important to note that, as clearly pointed out in (Rodrigue & Notteboom, 2013), customer choice of a cruise is based on the "overall appeal" of an itinerary. On one hand, this latter is certainly related to the attractiveness of the port cities visited, but on the other hand, it depends on the overall schedule of the itinerary and its operational conditions. Quoting from (Rodrigue & Notteboom, 2013): "*the cruise industry sells itinerary, not destinations*".

Therefore the Day–by–day Cruise Itinerary Optimization (DCIO) problem to be

[1]`https://www.seatrade-cruise.com`

solved in the third level of the decision process of a cruise company involves many different significant aspects to be taken into account. This makes the problem really challenging both from the modellistic and computational viewpoint. Nevertheless, literature dealing with quantitative methods in the cruise company decision making processes and, in particular, in itinerary design is very limited. In the systematic review reported in (Papathanassis & Backmann, 2011), the authors clearly evidence the scarcity of research on cruise shipping and this is attributed both to its domain's niche status and to a wide fragmentation due to the interdisciplinary nature of cruise studies. In particular, by observing the conclusions reported in Table 2 of (Papathanassis & Backmann, 2011), among the papers considered (published between 1983 and 2009) only 31% of them are quantitative research papers and only 6% are in the Engineering and Technology disciplinary domain. Even if this review refers to past years, anyhow in more recent years only few published papers actually use a mathematical approach, and in particular optimization techniques, for efficiently solving problems related to cruise shipping management. This is true nevertheless the great growth of cruise market observed in the last years led to an increase of the dimension and the complexity of problems in hand. Most of the recent literature on cruise shipping is focused on Economics and Business Management, namely marketing strategies, revenue management, demand analysis (see (Cusano, Ferrari, & Tei, 2017))). To confirm this, see also the recent book (Dowling & Weeden, 2017) which collects 35 papers providing a wide overview of the cruise industry covering a broad range of topics and issues. Even if the authors claim that the book has been written for a broad audience including planners and managers in the cruise industry, most of papers are focused on economic aspects (business models), environmental concerns (sustainable management), touristic issues (development of cruise tourism in particular regional areas), cruise safety and security (managing passenger health–related crisis). Until now, quantitative methods have been applied in maritime transport mainly dealing with freight transportation (see e.g. (Brouer, Karsten, & Pisinger, 2017; Gelareh & Pisinger, 2011)) or passenger ferries (see e.g. (J. Wang & McOwan, 2000)) rather than cruise sector.

In particular, few papers are devoted to mathematical modelling the day–by–day itinerary planning and to use optimization methods for solving DCIO problem. We mention the paper (Asta, Ambrosino, & Bartoli, 2018) where a Mixed Integer Linear Programming formulation of the DCIO problem is provided aiming at determining itineraries which maximize the revenues and customer satisfaction, and minimize the overall costs; (Cho, 2019) where an Integer Programming model is proposed as a reduced variant of the traveling salesman problem, aiming at maximizing passenger satisfaction; the paper (Mancini & Stecca, 2018) which proposes a model which is a variant of the vehicle routing problem along with a matheuristic which enables to efficiently solve large instances; (S. Wang, Wang, Zhen, & Qu, 2017) where the DCIO problem is solved by first enumerating all sequences of transit ports and then arrival and departure times are determined by using dynamic programming so that net profit is maximized; (Yang, Gao, & Li, 2016) where the authors developed a model for determining the maximum passenger volume with minimum operating costs by using a genetic algorithm.

Other papers on cruise itinerary design report results of empiric researches or are based heuristic approaches. See, e.g. (Lekaku, Pallis, & Vaggelas, 2009) where the authors focus on the selection of criteria to be used by a cruise company for deciding itinerary ports; (Leong & S.H. Ladany, 2001) where an heuristic approach is proposed and applied to instances from South–East area; the paper (Li, Wang, & Ducruet, 2020) which is limited to an analysis of the characteristic of the itineraries proposed by a world cruise company; (Santos, Martins, & Soares, 2021) where the authors show how the distribution of nautical distances between ports in Atlantic coast of the Iberian Peninsula and Mediterranean ports can be used for itinerary planning.

In this paper, we consider the DCIO problem aiming at determining the day–by–day itinerary in terms of transit ports and arrival and departure times, with the objective of minimizing the itinerary cost due to fuel and port costs, and maximizing an attractiveness index of the itinerary, related to the ports visited and the number of days spent at sea, i.e. without docking in a port. Therefore the problem turns out to be a bi–objective optimization problem. As to the first objective, the fuel consumption depends nonlinearly on the ship speed; the speed depends on the distances between the ports and the need to meet times for entering and leaving the port; the port cost depends on the port location and on the services provided. As to the second objective, it is evaluated by giving a rating to each port, to the days spent at sea (when the travel time between two successive ports exceeds 24 hours) and to overnights in port. Operational constraints are due to minimum and maximum number of transit ports to be visited, to the allowable time windows for arrival and departure in the port, to minimum and maximum time of stay in each port, to the fact that some ports may be obliged or prohibited, or may be visited only in given days, to minimum and maximum number of days spent at sea, to minimum and maximum number of ports where the ship moors at anchor and not at the dock. In particular, as we already mentioned, we refer to luxury cruises, implying several specific considerations to be taken into account, which lead to an increased difficulty of the problem.

We propose a Mixed Integer Linear Programming (MILP) model for this problem. We coded it by using AMPL language (Fourer, Gay, & Kernighan, 2003) and solved the resulting MILP problem by using a commercial solver. The solution gives the optimal day–by–day itinerary, in terms of ports to be visited and times of arrivals and departures, which satisfies all the operational constraints and locates a point of the Pareto frontier in the objective functions space. This model has been experimented by a major luxury cruise company to design the day–by–day itineraries of their cruises in many geographical areas all over the world. Here we report, as illustrative examples, the results obtained on some of these real instances to show the computational viability of the proposed approach. In this regard, we highlight that we adopt an exact solution approach, rather than the use of some metaheuristic, even if for some large instances this may lead to long computing times. This is motivated by the fact that itinerary planning is performed years in advance, so that even a long computing time for some instances is admissible. Of course, if the computing time exceeds a CPU time threshold value, the computational run can be early stopped, providing an approximate solution of the problem with the corresponding optimality gap, so that its accuracy can be assessed.

This work has been developed within a project named *Magellano Project*, a joint project between ACTOR SRL, a Start–Up of SAPIENZA University of Rome and a major luxury cruise company (which we do not mention for the sake of privacy). The overall project involves the three levels of the decision making process of the cruise company.

The paper is organized as follows: in Section 2 the description of the DCIO problem is reported. In Section 3 we describe in detail the mathematical model developed. In Sections 4 we describe how to use the model. In Section 5 we report some experimental results on real problem instances. Finally, some concluding remarks are drawn in Section 6.

2. Problem description

In this section we report all the elements that characterize the DCIO problem, with a particular focus on a cruise company that operates in the luxury market class. The problem data are: a ship, a maritime area, the turnaround ports, a time period (defined by starting and ending date of the itinerary) and a set of transit ports of touristic interest in the area. The turnaround ports are assumed to be selected in the ship deployment second level of the company decision process.

Observe that there are two kinds of ports: those where the ship can moor at the dock, and those where the ship moors only at anchor. This partition is a specific feature of luxury cruise companies, which usually operate with small tonnage ships, embarking only hundreds of passengers and not thousands, as happens for the mass market class. Indeed a ship of small tonnage can enter small ports of great touristic interest, by mooring at anchor and debarking passengers by motor boats, which is not possible if the number of passengers is too large.

The design of a cruise itinerary first consists in selecting the transit ports and their sequence. Usually, in an itinerary, a transit port is visited just once: only a turnaround port can be visited twice in the case the cruise starts and ends at that port. Each day no more than one port is visited, due to the time required by the maneuvers for mooring, the time required for debarking and embarking passengers and the free time spent on shore excursions by passengers. Therefore, usually a cruise ship arrives in a port in the morning and departs in the evening. However it can be the case that an overnight is spent in a port, if there is an event that motivates a longer stay or if the port (or some neighborhood) is of great touristic interest and shore excursions may last more than one day. In this case, the ship will depart in the evening of the next day. Moreover, it is possible that, between one port and the next one in the sequence, the ship sails for more than one day, without intermediate mooring; we call these days "days at sea" and they could be included in an itinerary when the distance between two ports is very long as, for instance, in oceanic cruises. However, this may happen also if the company considers fruitful to keep passengers on board, so that they spend on board money that otherwise would spend on shore.

The overall aim of the cruise itinerary planning is twofold: to determine the sequence of ports along with arrival and departure time, aiming at minimizing the cruise itinerary cost, while maximizing its attractiveness. It is important to note that, typically, as more attractive is a cruise itinerary, as more it costs, therefore the problem has conflicting objectives. In the sequel, we detail how the overall cost is computed on the basis of a number of compound costs and how the attractiveness of an itinerary is determined.

The *itinerary cost* is obtained as the sum of fuel cost and port costs; in turns, fuel consumption depends nonlinearly on the distance between ports and on the speed at which the distances are covered. The cruising speed between two ports depends on the distance between the ports and on the time windows for leaving and entering the ports, being these latter usually prefixed by port operators. The port cost depends on the maneuvering cost in arrival and departure, and on the cost of the stay in the port which is given by a fixed and an hourly component.

The *itinerary attractiveness* is obtained as follows. At each port of the area a *Port Attractiveness Index (PAI)* is assigned on the basis of the score obtained by evaluating a list of port attributes. For the sake of brevity, we do not report here the complete list, but we only mention the most representative: *overall perception* (general reputation, is iconic, political stability, safety); *port features* (port infrastructures, distance to city center); *interests and activities* (cultural interest, natural interest, food and beverage interest, shopping possibilities, shorex options/variety); *exclusivity* (crowding level, exclusive cruise destination). Moreover, an index is assigned to possible one *Days At Sea (DASI)*. This is evaluated by giving a score to interests and activities that can be proposed to the passengers on board. Finally, we consider another index related to possible one *Overnight In Port (OIPI)*. All these indices have been evaluated by means of the scores assigned by people from the cruise company marketing office involved in the project. Based on their expertise, they provided us with accurate answers to a specific questionnaire we proposed. Hence, we define the itinerary attractiveness as weighted sum of these three indices PAI, DASI and OIPI.

We now summarize the basic requests which must be considered when dealing with the mathematical formulation of the DCIO problem:

- C1: a port cannot be visited twice in a cruise itinerary, except the embark port;
- C2: some ports of the considered maritime area could be banned in given days;
- C3: the visit at some ports is mandatory, i.e. they must be included in the cruise itinerary;
- C4: the visit of some ports is mandatory in prefixed days;
- C5: an overnight in a given port in a given day is planned;
- C6: a given day of the cruise must be a day at sea;
- C7: the days at sea can not be consecutive.

Moreover, the following data must be specified when designing a cruise itinerary:

- B1: the minimum and maximum number of transit ports visited in the cruise itinerary;
- B2: the maximum number of anchor ports included in the cruise itinerary;
- B3: for each port, the time windows for arrival and departure time in the port;
- B4: the minimum and maximum time of stay in each port;
- B5: the maximum number of days at sea included in the cruise itinerary.

Note that some constraints are relevant to luxury cruises: in particular, B2 bounds the number of possible anchor ports, due to the discomfort of disembarkation and embarkation by motor boats; B5 bounds the number of days at sea and, along with C7, aims at avoiding that the cruise could become boring.

3. The mathematical model

In this section we describe the mathematical model we propose for solving the DCIO problem. In the sequel, we report all the elements of the model, i.e. the model input data, the decision variables, the objective functions and the constraints.

3.1. *The input data*

The input data are divided into two groups: the *scenario data*, that are common to all problem instances for the same ship in the same maritime area, and the *instance data* that are peculiar to a particular instance of the problem.

3.1.1. The scenario data

The scenario of the model is defined by the following data:

- the set $\mathcal{P}$ of the ports of interest in the maritime area;
- the set $\mathcal{P}_A \subset \mathcal{P}$ of the anchor ports;
- the set $\mathcal{V}$ of the (discretized) operating cruising speeds of the ship;
- for $p, q \in \mathcal{P}, v \in \mathcal{V}$, the time required for sailing from port p to port q at speed v denoted by $t(p, q, v)$;
- for $p, q \in \mathcal{P}, v \in \mathcal{V}$, the fuel cost for sailing from port p to port q at speed v, denoted by $c(p, q, v)$;
- for $p \in \mathcal{P}$, the fixed and the hourly cost of the stay in port p, denoted by $c_f(p)$ and $c_h(p)$, respectively;
- for $p \in \mathcal{P}$, the cost of departure and arrival maneuvering in port p, denoted by $cm_d(p)$ and $cm_a(p)$, respectively;
- for $p \in \mathcal{P}$, the time duration of departure and arrival maneuvering in port p, denoted by $tm_d(p)$ and $tm_a(p)$, respectively;
- for $p \in \mathcal{P}$, the starting and ending time for the arrival time window at port p, denoted by $atw_s(p)$ and $atw_e(p)$, respectively; therefore, the arrival time window is $[atw_s(p)\ ,\ atw_e(p)]$;
- for $p \in \mathcal{P}$, the starting and ending time for the departure time window at port p, denoted by $dtw_s(p)$ and $dtw_e(p)$ respectively; therefore the departure time window is $[dtw_s(p)\ ,\ dtw_e(p)]$;
- for $p \in \mathcal{P}$, the minimum and maximum stay time in port p, denoted by $minstay(p)$ and $maxstay(p)$, respectively;
- for $p \in \mathcal{P}$, the attractiveness index PAI of port p, denoted by $a(p)$;
- for $p \in \mathcal{P}$, the attractiveness index OIPI of one overnight at port p, denoted by $o(p)$;
- the attractiveness index DASI of one day at sea, denoted by ads.

Note that, of course both PAI and OIPI indices depend on port p, while the last defined index (DASI), being related to the whole itinerary, does not depend on ports.

3.1.2. The problem instance data

The following data characterize a particular instance of the problem:

- the ordered set $\mathcal{D} = \{0, \ldots, N\}$ of the days of the cruise itinerary; $d \in \mathcal{D}$ denotes a day of the cruise; the cruise itinerary starts at day $d = 0$ of the first embarkation and it ends at day $d = N$ of the last disembarkation;
- the embarkation and disembarkation turnaround ports of the cruise itinerary, denoted by $p^e \in \mathcal{P}$ and $p^d \in \mathcal{P}$, respectively; it may happen that p^e and p^d coincide;
- the set $\mathcal{P}_T = \mathcal{P} \setminus \{p^e, p^d\}$ of the transit ports of interest;
- the set $\mathcal{P}_V \subset \mathcal{P}$ of the ports that must be visited by the cruise;
- the minimum and maximum number of transit ports to be visited by the itinerary cruise, denoted by $npmin$ and $npmax$, respectively;
- the maximum number of anchor ports that can be visited by the itinerary cruise, denoted by $npmax_A$;
- the minimum and maximum number of days at sea allowed in the cruise itinerary, denoted by $mindas$ and $maxdas$, respectively;
- the set $\mathcal{D}_S \subset \mathcal{D}$ of days $\{d^i \mid d^i \in \mathcal{D}\}$, in which one day at sea must be planned, namely each day $d^i \in \mathcal{D}_S$ is such that more than 24 hours must be spent at sea, starting from the embarking on the day d^i;
- the set $\mathcal{M}_V \subset \mathcal{P} \times \mathcal{D}$ of couples $\{(p^i, d^i), p^i \in \mathcal{P}, d^i \in \mathcal{D}\}$ of ports p^i to be visited

on day d^i;

- the set $\overline{\mathcal{M}}_V \subset \mathcal{P} \times \mathcal{D}$ of couples $\{(p^i, d^i), p^i \in \mathcal{P}, d^i \in \mathcal{D}\}$ of ports p^i to be not visited on day d^i;

3.2. *The decision variables*

Now we introduce the decision variables of the model, that result to be both continuous and integer (binary), so that we have a mixed integer problem.

- $x(p, q, v)$ is a binary variable equal to 1 if in the itinerary cruise the ship covers the leg from port p to port q at speed v, equal to 0 otherwise;
- $y(p)$ is a binary variable equal to 1 is the ship visits the port p, equal to 0 otherwise
- $y_d(p, q, d)$ is a binary variable equal to 1 if the ship departs from port p headed towards port q on the day d, equal to 0 otherwise;
- $y_a(p, d)$ is a binary variable equal to 1 if the ship arrives in port p on day d;
- $das(p)$ is a binary variable equal to 1 if the ship arrives at port p having spent at least one day at sea, that is sailing for at least 24 hours after the departure from the preceding port; equal to 0 otherwise;
- $ovn(p)$ is a binary variable equal to 1 if the ship moors at port p at least 24 hours, that is an overnight in port p is planned; equal to 0 otherwise;
- $t_d(p, q)$ is a continuous variable denoting the departure time of the ship departing from port p towards port q; $t_d(p, q)$ is expessed in terms of hours and hundredths of hour, in the interval $\{0, \ldots, 24 \times N\}$, thus increasing with the days;
- $t_a(p)$ is a continuous variable denoting the arrival time of the ship in port p; $t_a(p)$ is expressed in terms of hours and hundredths of hour, in the interval $\{0, \ldots, 24 \times N\}$, thus increasing with the days;
- $t_s(p)$ is a continuous variable denoting the stay time of the ship in port p; $t_s(p)$ is expessed in terms of hours and hundredths of hour, in the interval $\{0, \ldots, 24 \times (1 + ovn(p))\}$.

3.3. *The objective functions*

We now report the expressions of the objective functions used in our formulation of the DCIO problem. They depend on input data and variables previously introduced. As described in Section 2, we consider two objective functions: the itinerary total cost (denoted by *cost*) to be minimized and the itinerary attractiveness (denoted by *attr*) to be maximized. Both objectives result from the sum of different components, that we report in the sequel.

3.3.1. The itinerary cost

The objective function value *cost* of an itinerary is given by

$$cost = fuelcost + staycost + mancost,$$

where

$$fuelcost = \sum_{p \in \mathcal{P}} \sum_{q \in \mathcal{P}} \sum_{v \in \mathcal{V}} c(p, q, v) x(p, q, v)$$

is the total cost of fuel,

$$staycost = \sum_{p \in \mathcal{P}} \Big(c_f(p)y(p) + c_h(p)t_s(p) \Big)$$

is the total cost for staying in the visited transit ports and

$$mancost = \sum_{p \in \mathcal{P}_T} cm_a(p)y(p) + cm_a(p^d) + \sum_{p \in \mathcal{P}} \sum_{q \in \mathcal{P}} \sum_{v \in \mathcal{V}} cm_d(p)x(p,q,v).$$

is the total cost for maneuvering in arrival and departure at the visited ports.

3.3.2. The itinerary attractiveness

The objective function value *attr* is given by

$$attr = attrpts + attrdas + attrovn,$$

where

$$attractpts = \sum_{p \in \mathcal{P}} a(p)y(p) \tag{3.1}$$

is the attractiveness component due to the visited ports,

$$attractdas = ads \sum_{p \in \mathcal{P}} das(p)$$

is the attractiveness component due to the days spent at sea and

$$attrovn = \sum_{p \in \mathcal{P}} o(p)ovn(p)$$

is the attractiveness component due to the overnights spent in ports. Note that in (3.1) p^e and p^d have been included in the sum even if they contribute with a constant term.

3.4. *The constraints*

In this section we describe the set of constraints which define the feasible set of the DCIO problem. They are subdivided into two groups: the *structural constraints* common to all problem instances, and the *operational constraints*, peculiar to a particular instance of the problem. In some constraints a parameter, denoted by $BigM$, is adopted to allow binary variables to turn constraints on or off.

3.4.1. Structural constraints

- Constraints ensuring that the cruise itinerary embarks and disembarks at the turnaround ports p^e, p^d:

$$y(p^e) = 1, \quad y(p^d) = 1.$$

- Constraint imposing that only one leg originates from the turnaround port p^e:

$$\sum_{q \in \mathcal{P}} \sum_{v \in \mathcal{V}} x(p^e, q, v) = 1.$$

- Constraint imposing that the ship departs from the turnaround port p^e on the first day of the cruise:

$$\sum_{q \in \mathcal{P}} y_d(p^e, q, 0) = 1.$$

- Constraints imposing that on the first day of the cruise itinerary the ship cannot depart from ports different from the embarkation turnaround port p^e:

$$\sum_{q \in \mathcal{P}} y_d(p, q, 0) = 0, \quad \text{for all} \quad p \in \mathcal{P}, \ p \neq p^e.$$

- Constraints imposing that the ship cannot depart from the turnaround port p^e on days different than the first day ($d = 0$) of the cruise itinerary:

$$\sum_{q \in \mathcal{P}} y_d(p^e, q, d) = 0, \quad \text{for all} \quad d \in \{1, \ldots, N\}.$$

- Constraints imposing that the ship cannot arrive to the turnaround port p^e on days different from the last day ($d = N$) of the cruise itinerary:

$$\sum_{p \in \mathcal{P}} y_d(p, p^e, d) = 0, \quad \text{for all} \quad d \in \{0, 1, \ldots, N-1\}.$$

- Costraints imposing that on the first day of the cruise itinerary ($d = 0$) the ship can not arrive to any port:

$$y_a(p, 0) = 0, \quad \text{for all} \quad p \in \mathcal{P}.$$

- Constraint imposing that the there is only one leg leading to the turnaround port p^d:

$$\sum_{p \in \mathcal{P}} \sum_{v \in \mathcal{V}} x(p, p^d, v) = 1.$$

- Constraints imposing that on the last but one day of the cruise itinerary ($d = N - 1$) the ship can not depart towards any port different from the turnaround port p^d:

$$\sum_{p \in \mathcal{P}} y_d(p, q, N-1) = 0, \quad \text{for all} \quad q \in \mathcal{P}, q \neq p^d.$$

- Constraints imposing that the ship cannot depart from the turnaround port p^d unless $p^e = p^d$ and $d = 0$, first day of the cruise itinerary:

$$\sum_{q \in \mathcal{P}} y_d(p^d, q, d) = 0, \quad \text{for all} \quad d \in \mathcal{D}, d \neq 0.$$

- Constraints imposing that the ship cannot depart on the last day $d = N$:

$$\sum_{q \in \mathcal{P}} y_d(p, q, N) = 0, \quad \text{for all} \quad p \in \mathcal{P}.$$

- Constraints that set the variables $y(p)$ to 1 if the port p with $p \neq p^e$ is visited, to 0 otherwise:

$$y(p) = \sum_{q \in \mathcal{P}} \sum_{v \in \mathcal{V}} x(q, p, v), \quad \text{for all} \quad p \in \mathcal{P}, \quad p \neq p^e.$$

- Constraints relating the variables $y_a(p, d)$ and $x(q, p, v)$:

$$\begin{aligned} \sum_{d \in \mathcal{D}} y_a(p, d) &= \sum_{q \in \mathcal{P}} \sum_{v \in \mathcal{V}} x(q, p, v), \quad \text{for all} \quad p \in \mathcal{P}, \\ y_a(p^d, N) &= \sum_{q \in \mathcal{P}} \sum_{v \in \mathcal{V}} x(q, p^d, v). \end{aligned}$$

- Constraints on the variables $y(p, q, d)$ imposing that in any given port, in any given day, at most one departure is possible:

$$\sum_{q \in \mathcal{P}} y_d(p, q, d) \leq 1, \quad \text{for all} \quad p \in \mathcal{P} \quad \text{and for all} \quad d \in \mathcal{D}.$$

- Constraints imposing that any given port cannot be visited more than once during the cruise itinerary:

$$\sum_{d \in \mathcal{D}} y_a(p, d) \leq 1, \quad \text{for all} \quad p \in \mathcal{P}.$$

- Constraints defining the arrival time of the ship in port p:

$$t_a(p) = \sum_{q \in \mathcal{P}} t_d(q, p) + \sum_{q \in \mathcal{P}} \sum_{v \in \mathcal{V}} \Big(t(q, p, v) + tm_d(q) + tm_a(p) \Big) x(q, p, v), \text{ for all} \quad p \in \mathcal{P}.$$

Note that, by this constraint, it results $t_a(p) = 0$ if the port p is not visited.

- Constraints defining the stay time of the ship in port p:

$$\begin{aligned} t_s(p^e) &= 0, \qquad t_s(p^d) = 0, \\ t_s(p) &= \sum_{q \in \mathcal{P}} t_d(p,q) - tm_a(p), \quad \text{for all} \quad p \in \mathcal{P}_T. \end{aligned}$$

- Sequencing constraints on the departure times:

$$\begin{aligned} & t_d(p,q) + \sum_{q \in \mathcal{P}} \sum_{v \in \mathcal{V}} (tm_d(p) + t(p,q,v) + tm_a(q)) x(p,q,v) - \sum_{r \in \mathcal{P}} t_d(q,r) \\ & \leq BigM \Big(1 - \sum_{q \in \mathcal{P}} \sum_{v \in \mathcal{V}} x(p,q,v) \Big), \quad \text{for all} \quad p \in \mathcal{P}_T \quad \text{and for all} \quad q \in \mathcal{P}_T. \end{aligned}$$

- Continuity constraints:

$$\sum_{p \in \mathcal{P}} \sum_{v \in \mathcal{V}} x(p,q,v) = \sum_{r \in \mathcal{P}} \sum_{v \in \mathcal{V}} x(q,r,v), \quad \text{for all} \quad q \in \mathcal{P}_T.$$

- Constraints required if the set $\mathcal{D}_S$ is not empty, i.e. if at least one day at sea is planned:

$$\sum_{p \in \mathcal{P}} \sum_{q \in \mathcal{P}} y_d(p,q,d^i) = 1, \quad \sum_{p \in \mathcal{P}} y_a(p, d^i + 1) = 0, \quad \text{for all} \quad d^i \in \mathcal{D}_S.$$

  These constraints ensure that if the ship departs from some port on day $d^i \in \mathcal{D}_S$, it does not arrive in any port on day $(d^i + 1)$.

- Constraints imposing that the departure time of the ship occurs within the departure time window:

$$\sum_{d \in \{0,\ldots,N-1\}} (dtw_s(p) + 24d) y_d(p,q,d) \leq t_d(p,q) \leq \sum_{d \in \{0,\ldots,N-1\}} (dtw_e(p) + 24d) y_d(p,q,d),$$

  for all $p, q \in \mathcal{P}$. Note that by this constraint it results $t_d(p,q) = 0$ if the leg (p,q) does not belong to the itinerary.

- Constraints imposing that the arrival time of the ship occurs within the arrival time window:

$$\sum_{d \in \{1,\ldots,N\}} (atw_s(p) + 24d) y_a(p,d) \leq t_a(p) \leq \sum_{d \in \{1,\ldots,N\}} (atw_e(p) + 24d) y_a(p,d),$$

  for all $p \in \mathcal{P}$.

- Constraints on the minimum and maximum stay time in each port:

$$minstay(p) \sum_{q \in \mathcal{P}} \sum_{v \in \mathcal{V}} x(p,q,v) \leq t_s(p) \leq maxstay(p) \sum_{q \in \mathcal{P}} \sum_{v \in \mathcal{V}} x(p,q,v),$$

  for all $p \in \mathcal{P}_T$.

- Constraint that sets the variable $das(p) = 1$ if the ship arrives at port p after sailing for more than 24 hours (day at sea), and sets $das(p) = 0$ otherwise:

$$-BigM\ (1 - das(p)) \leq \sum_{q \in \mathcal{P}} \sum_{v \in \mathcal{V}} t(p,q,v)x(p,q,v) - 24.01 \leq BigM\ das(p). \quad (3.2)$$

- Constraint that sets the variable $ovn(p) = 1$ if the ship moors at port p for more than 24 hours (overnight in port), and set $ovn(p) = 0$ otherwise:

$$-BigM\ (1 - ovn(p)) \leq t_s(p) - 24.01 \leq BigM\ ovn(p), \quad \text{for all} \quad p \in \mathcal{P}. \quad (3.3)$$

3.4.2. Operational contraints

- Constraint on the minimum and maximum number of transit ports included in the cruise itinerary:

$$npmin \leq \sum_{p \in \mathcal{P}_T} y(p) \leq npmax. \quad (3.4)$$

- Constraint on the transit ports to be visited in the cruise itinerary:

$$y(p) = 1 \quad \text{for all} \quad p \in \mathcal{P}_V.$$

- Constraint on the transit ports to be visited in given arrival days in the cruise itinerary:

$$y_a(p,d) = 1 \quad \text{for all} \quad (p,d) \in \mathcal{M}_V. \quad (3.5)$$

- Constraint on the ports to be not visited in given arrival days in the cruise itinerary:

$$y_a(p,d) = 0 \quad \text{for all} \quad (p,d) \in \overline{\mathcal{M}}_V. \quad (3.6)$$

- Constraint on the maximum number of anchor ports included in the cruise itinerary:

$$\sum_{p \in \mathcal{P}_A} y(p) \leq npmax_A. \quad (3.7)$$

- Constraints on the minimum and maximum number of days at sea in the cruise itinerary:

$$mindas \leq \sum_{p \in \mathcal{P}} das(p) \leq maxdas.$$

As concerns the constraints (3.4) on the number of visited transit ports, by letting $npmin = npmax = N - 1$ it is possible to impose that at each day a port is visited, thus avoiding days at sea and/or overnights in port. Instead, by setting $npmin < npmax < N - 1$ more freedom is given for days at sea and/or overnights in port. The constraints (3.5) on the transit port to be visited is self explanatory and it is usually refereed to a port of great attractiveness: if the port is to be visited in a given day, it means that some event occurs in this day which could be of great interest for the cruise passengers. The constraints (3.6) on the transit ports to be not visited in given days is typically adopted when some shore activities are not allowed in these ports in these days; as an example usually cruise ships do not moors in Civitavecchia (the port of Rome) on Monday, when in Rome all museums are closed. Of course, note that to impose that a ports has not to be not visited in the whole cruise itinerary it is enough to remove these ports from the set of ports $\mathcal{P}$. The constraints (3.7) on the maximum number of allowed anchor ports in the cruise itinerary is self explanatory, too: even if an anchor port is of great touristic interest, disembarking and embarking by motor boats would result somewhat uncomfortable for the cruise passengers. The constraints (3.3) on the stay time allows for one overnight in port p if $y(p) = 1$ and $minstay(p) \geq 24$. Finally, by constraints (3.2), one day d^i at sea is forced if $\sum_{p\in\mathcal{P}} das(p) = 1$ and

$$\sum_{p\in\mathcal{P}}\sum_{q\in\mathcal{P}} y_d(p,q,d^i) = 1, \qquad \sum_{p\in\mathcal{P}} y_a(p,d^i+1) = 0;$$

of course, in this case, it is required that the set $\{t(p,q,v), p,q \in \mathcal{P}, v \in \mathcal{V}\}$ contains enough time legs $t(p,q,v) \geq 24$.

3.5. *The Bi–Objective Mixed Integer Linear Programming problem*

We can now formulate the DCIO problem as the optimization problem aiming at minimizing the objective function *cost*, while maximizing the objective function *attr* defined in Section 3.3. Therefore, if we denote by $\mathcal{Z}$ the feasible set of the problem, i.e. the set of decision variables which satisfy all the constraints, then the DCIO problem can be stated as the following optimization problem

$$\begin{aligned} &\min\Big\{cost(z), -attr(z)\Big\} \\ &\text{subject to } z \in \mathcal{Z}. \end{aligned} \tag{3.8}$$

Recalling that the decision variables are both continuous and integer (binary) and observing that both objective functions and all the constraints are linear, problem (3.8) is a *Mixed Integer Linear Programming (MILP)* problem. Moreover, based on the fact that, as we already observed, the more an itinerary is attractive, the higher its cost is, problem (3.8) has conflicting objectives, i.e. it is a nontrivial bi–objective optimization problem (for an extensive treatment on Multiobjective Optimization see, e.g.(Miettinen, 2012)).

3.6. *Some remarks on the optimization model*

We remark that, for the sake of simplicity, the model presented here is a simplified version of the one actually developed. In particular, as an example, some of the features that have been omitted are the following:

- to impose that the visited ports belongs to different countries, thus avoiding to pay the “cabotage tax” (tax concerning rights of a company from one country to trade in a different country);

- to consider two (or more) consecutive days at sea, with decreasing attractiveness; this is of interest, in particular, for oceanic cruises;
- to consider more than one overnight in a port; this is of interest for events lasting more than one day;
- to prevent that anchor ports are visited in consecutive days;
- to consider more than one transit port in the same day, even if this is of interest only for cruises in the class of expeditions;
- for some given ports $p \in \mathcal{P}$, to consider the departure time window $[dtw_s(p), dtw_e(p)]$ and the arrival time window $[atw_s(p), atw_e(p)]$ split on two successive days.

It is clear that including in the description of the model all the additional features would require much more room, while they are not strictly necessary for understanding the approach we propose for tackling the cruise itinerary optimization problem.

4. The optimization procedure

We now describe the optimization procedure leading to the design of the cruise itinerary which minimize the overall cost, while maximizing its attractiveness, namely to solve the MILP problem (3.8). To this aim, let us denote by $\mathcal{F}$ the two dimensional *objective functions space* $(cost, attr)$. As well known, a solution $z^* \in \mathcal{Z}$ of problem (3.8) is a point that maps itself on the *Pareto frontier* or *efficient frontier* in $\mathcal{F}$. The latter is defined as the set of points for which any feasible movement from z^* which improves the function *cost* on $\mathcal{F}$, worsen the function *attr* on $\mathcal{F}$ and conversely. Moreover, we consider the *ideal objective values* $cost^*$ and $attr^*$, i.e. the optimal values of the two single objective optimization problems resulting by considering separately the two objective functions, namely

$$\begin{array}{ll} \min & cost(z) \\ \text{s.t.} & z \in \mathcal{Z} \end{array} \tag{4.1}$$

and

$$\begin{array}{ll} \max & attr(z) \\ \text{s.t.} & z \in \mathcal{Z}. \end{array} \tag{4.2}$$

Furthermore, let us denote by z_c^* and z_a^* the optimal points of problems (4.1) and (4.2), respectively, namely points belonging to $\mathcal{Z}$ such that it results $cost^* = cost(z_c^*)$ and $attr^* = attr(z_a^*)$. Then we define $attr^{\#} = attr(z_c^*)$ and $cost^{\#} = cost(z_a^*)$.

As well known, the two methods elective for locating points of the Pareto frontier of a bi–objective optimization problem are the *method of weights* and the *method of constraints* (see (Miettinen, 2012)). In the *method of weights* (also called *scalarization method*) the bi–objective problem is transformed into the single objective optimization problem whose objective function is the weighted sum of the two objective functions. Namely, it consists in solving the problem

$$\begin{array}{ll} \min & (1-w) \cdot cost(z) - w \cdot attr(z) \\ \text{s.t.} & z \in \mathcal{Z}, \end{array} \tag{4.3}$$

where $w \in [0, 1]$ is a suited weight. Of course, setting $w = 0$ we get the objective function values $cost^*, attr^\#$, while setting $w = 1$ we get the values $cost^\#, attr^*$. By letting $w \in (0, 1)$ we get points on the Pareto frontier.

The *method of constraints* consists in the minimization of one objective function and considering the other objective as constraint, imposing that it is bounded by some threshold value. Namely, the following two problems can be considered

$$\begin{array}{rl} \min & cost(z) \\ \text{s.t.} & z \in \mathcal{Z} \\ & attr(z) \geq \overline{attr} \end{array} \tag{4.4}$$

and

$$\begin{array}{rl} \max & attr(z) \\ \text{s.t.} & z \in \mathcal{Z} \\ & cost(z) \leq \overline{cost}, \end{array} \tag{4.5}$$

where the bounds $\overline{attr}$ and $\overline{cost}$ are such that

$$attr^\# \leq \overline{attr} \leq attr^* \qquad \text{and} \qquad cost^* \leq \overline{cost} \leq cost^\#. \tag{4.6}$$

In both cases, by varying the values $\overline{attr}$ and $\overline{cost}$, we get points of the Pareto frontier.

5. Experimental results on some illustrative instances

The approach we propose in this paper has been experimented by a luxury cruise company for defining cruise itineraries for different ships located in various geographical areas all over the world. Of course, here we have no room for reporting results concerning the whole set of itineraries of the areas considered. We report the results obtained on some instances in order to show the reliability of our approach and also its computational viability. As regards the latter issue, we highlight that the DCIO problem results to be a large scale MILP problem, whose dimension increases with the cardinality of the sets $\mathcal{P}$ and $\mathcal{D}$, i.e. the number of ports considered and the duration of the itineraries. This could lead to long computing time needed to solve the problem. However, in the framework of the decision making process of a cruise company, computing time does not represent a crucial issue, since itinerary design is performed a long time in advance, usually a couple of years. This motivated us to use a commercial MILP solver (rather than adopt a metaheuristic solution approach), possibly stopping solver iterations when the relative optimality gap is below a prefixed threshold value.

We coded the MILP model for the DCIO problem by using AMPL language (Fourer et al., 2003) and we used the GUROBI 9.1 solver (*GUROBI Optimizer reference manual*, 2020). All the runs have been performed on a PC with an Intel Core i7-2600 3.40 GHz Processor and 16 GB RAM. The runs were stopped when the relative optimality gap satisfies $rel\_opt\_gap \leq 0.05$. As regards the parameter $BigM$ used in the definition of some constraints, we set $BigM = 10^6$.

As illustrative example of a real instance of the DCIO problem we consider cruises in the West Mediterranean maritime area, embarking at Barcelona (Spain), disembarking at Civitavecchia, the port of Rome (Italy), and lasting 7 days. The set of transit ports includes 106 ports of Spain, France, Monte Carlo, and Italy located in this maritime area. As regards the port names, in the sequel we adopt the standard abbreviation from the *United Nations Code for Trade and Transport Locations* (UN/LOCODE Code List 2020-2)[2], consisting in a combination of a 2-character country code and a 3-character location code (e.g. ESBCN stands for Barcelona, Spain).

All the scenario data listed in Section 3.1.1 which refer to each port $p \in \mathcal{P}$ are given. The set of speeds $\mathcal{V}$ covers the operational interval $[10, 18]$ nautical miles/hour, discretized by a 0.5 step. The arrival time window is set to [06:00 , 10:00] a.m. and the departure time window to [06:00 , 10:00] p.m. As regards the stay time in port we set $minstay = 8$ hours and $maxstay = 16$ hours. Finally the index DASI is given.

As regards the problem instance data reported in Section 3.1.2, we have the following ones common to all the instances reported in the sequel: $N = 7$ in the definition of $\mathcal{D}$, the turnaround ports p^e=ESBCN and p^d=ITCVV, the set of transit port $\mathcal{P}_T$ (which is not reported extensively for the sake of brevity) whose cardinality is 106, $npmax_{\mathcal{A}} = 1$, $mindas = 0$, $maxdas = 1$. All the remaining problem data, namely $npmin$, $npmax$, the sets $\mathcal{D}_S$, $\mathcal{M}_V$ and $\overline{\mathcal{M}}_V$, are specified in correspondence of each instance. By default, we assume: $\mathcal{P}_V = \emptyset$, $\mathcal{D}_S = \emptyset$ and $\mathcal{M}_V = \overline{\mathcal{M}}_V = \emptyset$.

The results are reported in terms of values of the objectives $cost$ and $attr$ at the optimal solution, namely corresponding the feasible solution which satisfy the prefixed optimality gap; the computing time (in seconds) required to get such a solution. The list of the legs of the cruise itinerary[3], the speed at which each leg is traveled (in nautical miles/hour), the voyage time required (in hours), the duration of the arrival maneuver (in hours), the arrival hour (within the 24 hours), the stay time at port (in hours), the departure hour (within the 24 hours), the duration of the departure maneuver (in hours), the index denoting if the port of arrival is a dock (D) or an anchor port (A), and the attractiveness of the leg (i.e. related to the arrival port or to a day at sea or to an overnight at port). Fractions of hour are expressed in hundredths. All cost and attractiveness values are scaled by a factor which is unspecified to protect strategic corporate data.

5.1. *Instance 1: minimization of the cost*

In the first instance we consider $npmin = npmax = 6$, $mindas = maxdas = 0$, aiming at minimizing the overall itinerary cost, namely we solve Problem (4.1) (or equivalently Problem 4.3 with $w = 0$). Table 5.1 reports the results obtained along with the ideal value of the cost ($cost^*$) and the corresponding value of attractiveness ($attr^\#$). It can be observed as, since the aim is to minimize the itinerary cost, the corresponding overall attractiveness is relatively low, due to the inclusion in the selected itinerary of ports with a low PAI (see e.g. ITSML and ITPTO)

5.2. *Instance 2: maximization of the attractiveness*

In the second instance, analogously to the first one, we consider $npmin = npmax = 6$, $mindas = maxdas = 0$ but we aim at maximizing the itinerary attractiveness, namely we solve Problem (4.2), (or equivalently Problem 4.3 with $w = 1$). Table 5.2 reports the results obtained along with the ideal value of the attractiveness ($attr^*$) and the

[2]`https://unece.org/trade/cefact/unlocode-code-list-country-and-territory`

[3]For the sake of clearness, the explicit names of the ports listed in the tables which follow, are reported here. Ports of Spain: Barcelona (ESBCN), Porto Mahon (ESMAH), Palma de Mallorca (ESPMI), Valencia (ESVLC), Spain; ports of France: Ajaccio (FRAJA), Hyères (FRHYR), Marseille (FRMRS), Porto–Vecchio (FRPVO); ports of Italy: Alghero (ITAHO), Civitavecchia (ITCVV), Livorno (ITLIV), Portofino (ITPTF), Porto Torres (ITPTO), Santa Margherita Ligure (ITSML); port of Monaco: Monte Carlo (MCMCM).

	Port from	*Port to*	*Speed m/h*	*Voy. time*	*Arr. man.*	*Arr. hour*	*Stay time*	*Dep. hour*	*Dep. man.*	*D A*	*Attr*
0		ESBCN						18.00	1.00	D	57
1	ESBCN	FRHYR	16.0	13.94	1.00	9.94	8.06	18.00	1.00	D	44
2	FRHYR	ITSML	12.0	14.00	1.00	10.00	8.44	18.44	1.00	D	42
3	ITSMG	FRAJA	13.5	13.56	1.00	10.00	10.08	20.08	1.00	D	51
4	FRAJA	ITAHO	12.0	7.92	1.00	6.00	16.00	22.00	1.00	A	45
5	ITAHO	ITPTO	12.0	6.67	1.00	6.67	15.33	22.00	1.00	D	42
6	ITPTO	FRPVO	12.0	6.33	1.00	6.33	12.25	18.58	1.00	D	45
7	FRPVO	ITCVV	12.0	9.42	1.00	6.00				D	53

$rel\_opt\_gap = 0.0404$, $cost^* = 190,253.10$, $attr^\# = 379$, CPU elapsed time=63.36

Table 5.1. Optimal cruise itinerary for the *Instance 1: minimization of the cost*

	Port from	*Port to*	*Speed m/h*	*Voy. time*	*Arr. man.*	*Arr. hour*	*Stay time*	*Dep. hour*	*Dep. man.*	*D A*	*Attr*
0		ESBCN						18.00	1.00	D	57
1	ESBCN	ESPMI	11.0	12.00	1.00	8.00	11.76	19.76	1.00	D	58
2	ESPMI	ESVLC	17.0	8.24	1.00	6.00	12.29	18.29	1.00	D	55
3	ESVLC	ESMAH	17.0	13.71	1.00	10.00	9.47	19.47	1.00	D	57
4	ESMAH	FRMRS	17.0	12.53	1.00	10.00	8.00	18.00	1.00	D	55
5	FRMRS	ITPTF	17.0	12.29	1.00	8.29	11.89	20.18	1.00	A	55
6	ITPTF	MCMCM	11.0	7.82	1.00	6.00	12.00	18.00	1.00	D	56
7	MCMCM	ITCVV	17.0	12.71	1.00	8.71				D	53

$rel\_opt\_gap = 0.0470$, $attr^* = 446$, $cost^\# = 288,534.38$, CPU elapsed time=38.91

Table 5.2. Optimal cruise itinerary for the *Instance 2: maximization of the attractiveness*

corresponding value of the cost ($cost^\#$). We can note that, as expected, all ports selected in this itinerary have a high attractiveness (greater than or equal to 53) and the overall attractiveness is 446 which is significantly greater than the one obtained in Instance 1. Of course, the corresponding itinerary cost is significantly increased, too.

5.3. *Instance 3: maximization of attractiveness and cost* $\leq \overline{cost}$

Now we consider the same data of Instance 2 but, taking into account that the value of the objective *cost* is bounded from below by the values $cost^* = 190,253.10$ and from above by the value $cost^\# = 288,534.38$, we now solve Problem (4.5), aiming at maximizing the attractiveness, while imposing that cost does not exceed the value $\overline{cost} = 250,000.00$ (which is an intermediate value between $cost^*$ and $cost^\#$, see (4.6)). Table 5.3 reports the results obtained along with the optimal value of the attractiveness ($attr^*$) and the corresponding value of the cost. It can be observed as, with respect to Instance 2 the loss in the attractiveness value amounts to only 4 points, while the saving in cost amounts to 42,880.20.

5.4. *Instance 4: visit of a particular port* (ITLIV)

In the preceding instances the port ITLIV is not included in any itinerary. However, since ITLIV (Livorno, Italy) is the basis for shore excursions to Pisa and Florence, both of great touristic interest, it could be important to impose that ITLIV is a port visited during the cruise itinerary. Therefore, we now use the same data of the previous instances, but we set $\mathcal{P}_V = \{\text{ITLIV}\}$. Moreover, we now search for a point on the Pareto

	Port from	*Port to*	*Speed m/h*	*Voy. time*	*Arr. man.*	*Arr. hour*	*Stay time*	*Dep. hour*	*Dep. man.*	*D A*	*Attr*
0		ESBCN						18.00	1.00	D	57
1	ESBCN	ESPMI	12.0	11.00	1.00	7.00	12.50	19.50	1.00	D	58
2	ESPMI	ESMAH	12.0	8.50	1.00	6.00	12.00	18.00	1.00	D	57
3	ESMAH	FRMRS	16.5	12.91	1.00	8.91	9.09	18.00	1.00	D	55
4	FRMRS	MCMCM	12.0	10.67	1.00	6.67	14.16	20.83	1.00	D	56
5	MCMCM	ITPTF	12.0	7.17	1.00	6.00	12.00	18.00	1.00	A	55
6	ITPTF	FRAJA	13.0	13.93	1.00	9.93	8.02	18.00	1.00	D	51
7	FRAJA	ITCVV	13.0	13.39	1.00	9.39				D	53

$rel\_opt\_gap = 0.0497$, $attr^* = 442$, $cost = 245,645.18$, CPU elapsed time=134.12

Table 5.3. Optimal cruise itinerary for the *Instance 3: maximization of the attractiveness while imposing* $cost \le \overline{cost} = 250,000.00$

frontier of the bi–objective problem, hence we consider Problem (4.3) where the weight w is chosen as the tradeoff value $w = 0.5$, thus balancing the two objectives $cost$ and $attr$. Table 5.4 reports the results for this instance. We can see that, as requested,

	Port from	*Port to*	*Speed m/h*	*Voy. time*	*Arr. man.*	*Arr. hour*	*Stay time*	*Dep. hour*	*Dep. man.*	*D A*	*Attr*
0		ESBCN						18.00	1.00	D	57
1	ESBCN	FRHYR	16.0	13.94	1.00	9.94	8.06	18.00	1.00	D	44
2	FRHYR	FRAJA	12.0	11.75	1.00	7.75	12.33	20.08	1.00	D	51
3	FRAJA	ITAHO	12.0	7.92	1.00	6.00	16.00	22.00	1.00	A	45
4	ITAHO	ITPTO	12.0	6.67	1.00	6.67	15.00	21.67	1.00	D	42
5	ITPTO	FRPVO	12.0	6.33	1.00	6.00	12.00	18.00	1.00	D	45
6	FRPVO	ITLIV	12.0	10.83	1.00	6.83	11.34	18.17	1.00	D	53
7	ITLIV	ITCVV	12.0	9.83	1.00	6.00				D	53

$rel\_opt\_gap = 0.0457$, $attr = 390$, $cost = 201,834.33$, CPU elapsed time=30.64

Table 5.4. Optimal cruise itinerary for the *Instance 4: Problem* (4.3) *with* $w = 0.5$ *and* $\mathcal{P}_V = \{\text{ITLIV}\}$

the port ITLIV is visited during the cruise itinerary. In particular, this occurs on the 6th day of the itinerary, on the way to the final turnaround port ITCVV. However, to impose that ITCVV is among the visited ports leads to a decrease of the itinerary attractiveness since ports with a high PAI are not be included to avoid high cost ports.

5.5. *Instance 5: one overnight in a particular port* (MCMCM)

Monte Carlo is well known for the motor race Grand Prix of Monaco, that will be disputed on May 22–23, 2021. The next case considers the same problem of Instance 2, which aims at maximizing the itinerary attractiveness (Problem 4.3 with $w = 1$), but now we add the request of an overnight in the port MCMCM on May 22, the 3rd day of the itinerary, in order to allow the cruise passengers to attend the race. Of course in this case the minimum stay time in MCMCM has been changed to 24 hours and the maximum to 48 hours. Note that in this case we have $npmin = npmax = 5$ rather than 6, as in all previous cases. In Table 5.5 the results for this instance are reported. Observe that, as requested, one overnight at MCMCM port is scheduled in the 4th day of the itinerary and this affects the overall attractiveness of the itinerary cruise. In fact, a slight decrease of the attractiveness is obtained with respect to the optimal cruise itinerary corresponding to Instance 2 (from which this Instance 5 is derived).

	Port from	*Port to*	*Speed m/h*	*Voy. time*	*Arr. man.*	*Arr. hour*	*Stay time*	*Dep. hour*	*Dep. man.*	*D A*	*Attr*
0		ESBCN						18.00	1.00	D	57
1	ESBCN	ESMAH	13.5	10.07	1.00	6.07	13.40	19.47	1.00	D	57
2	ESMAH	FRMRS	17.00	12.53	1.00	10.00	12.00	22.00	1.00	D	55
3	FRMRS	MCMCM	13.5	9.48	1.00	9.48	36.52	22.00	1.00	D	56
4					Overnight at MCMCM						56
5	MCMCM	ITPTF	14.0	6.14	1.00	6.14	11.86	18.00	1.00	A	55
6	ITPTF	FRAJA	16.0	11.31	1.00	7.31	13.46	20.77	1.00	D	51
7	FRAJA	ITCVV	15.5	11.23	1.00	10.00				D	53

rel_opt_gap = 0.0500, $attr^* = 440$, $cost^\# = 228,212.62$, CPU elapsed time=80.85

Table 5.5. Optimal cruise itinerary for the *Instance 5: Problem* (4.3) *with* $w = 1$, *imposing one overnight in* MCMCM *port*

5.6. *Instance 6: one day at sea in the 5th day of the itinerary*

As last instance we consider an itinerary with a day at sea, starting with the departure in the evening on the 4th day and lasting during the whole 5th day. Of course, again we have $npmin = npmax = 5$. Taking into account that, due to the reduced number of ports, we expect a reduced port cost and reduced attractiveness, analogously to the previous instance, we consider Problem 4.3 with $w = 1$, so that the itinerary attractiveness is maximized. Table 5.6 reports the results obtained for this instance.

	Port from	*Port to*	*Speed m/h*	*Voy. time*	*Arr. man.*	*Arr. hour*	*Stay time*	*Dep. hour*	*Dep. man.*	*D A*	*Attr*
0		ESBCN						18.00	1.00	D	57
1	ESBCN	FRMRS	17.0	10.82	1.00	6.82	11.18	18.00	1.00	D	55
2	FRMRS	ESMAH	15.5	13.81	1.00	9.81	8.19	18.00	1.00	D	57
3	ESMAH	ESVLC	17.0	13.71	1.00	9.71	12.29	22.00	1.00	D	55
4	ESVLC	ESPMI	17.0	8.24	1.00	8.24	12.85	21.09	1.00	D	58
5					Day at sea						20
6	ESPMI	MCMCM	11.0	34.91	1.00	10.00	8.00	18.00	1.00	D	56
7	MCMCM	ITCVV	17.0	12.71	1.00	8.71				D	53

rel_opt_gap = 0.0000, $attr^* = 411$, $cost^\# = 286,160.77$, CPU elapsed time=398.19

Table 5.6. Optimal cruise itinerary for the *Instance 6: Problem* (4.3) *with* $w = 1$, *imposing one day at sea at 5th day of the itinerary*

Of course, due to the low attractiveness which has been assigned in this case to the day at sea (20), the overall itinerary attractiveness is decreased with respect to that obtained in the previous Instance 5, even if its cost is significantly increased.

In order to give evidence of the large dimension of the problem in hand, we report the number of variables and the number of constraints which characterize the instances now considered: problems have typically around 62,600 variables of which around 2,900 are continuous and the remaining are binary; the number of constraints varies between 18,850 and 22,040. Of course, the problem dimension increases as the number of ports considered $|\mathcal{P}|$ and the itinerary duration $|\mathcal{D}|$ increase. Therefore, solving the MILP problems corresponding to large instances could require a long computing time. However, as we already pointed out, usually this does not represent a serious drawback since the itinerary planning is designed a long time in advance.

6. Conclusions

This paper deals with the day–by–day cruise itinerary planning for a ship operating in a given maritime area. It represents the lowest level of the decision making process of a cruise company in designing cruises to propose to their customers. In particular, we focused on luxury cruise market for which additional constraints must be taken into account, with respect to cruise mass market. Actually, the problem has a twofold objective: from one hand the aim is to minimize the overall itinerary cost and, from the other hand, the itinerary attractiveness should be maximized. These are two conflicting objectives, since the more attractive the itinerary, the higher the cost. Therefore, we formulated the DCIO problem as a bi–objective MILP problem. Actually, for the sake of clearness, in the paper a simplified version of the model experimented by a luxury cruise company is reported. However, the main features of the proposed approach can still be observed, avoiding discussions on technical details regarding additional specific requests.

As illustrative example, we report the optimal cruise itineraries for some instances in the West Mediterranean maritime areas. We highlight that the model has been used for defining cruise itineraries in many different geographical areas all over the world, considering many different parameter settings and particular additional requests. We showed that the model we propose allows the user to obtain the cruise optimal itinerary by using a commercial MILP solver. Of course, the optimization model we propose is to be intended as a decision support system for the company management, who is the only one deputed to refine, improve and finalize the cruise itineraries into the cruise catalog actually proposed to customers.

We believe that the revival of the cruise industry after the COVID-19 crisis should be also based on decision support systems like the one proposed in this paper, aiming at designing new and more attractive cruise itineraries, trying to contain costs.

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