A New Method for the Design Optimization of Three-Phase Induction Motors

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Abstract - **The paper deals with the optimization problem of induction motors design. In particular a new global minimization algorithm is described : it tries to take into account all the features of these particular problems. A first numerical comparison between this new algorithm and a method widely used in the design optimization of induction motors has been performed. The obtained results show that the proposed approach is promising.**

Index terms - **Global optimization, derivative-free methods, induction motors.**

I. Introduction

The design of induction motors is based on universally accepted physical and mathematical principles which have been verified by experimental methods. The knowledge of these principles is often insufficient to produce the "optimal design" due to the fast changing technological developments.

Moreover, with the increasing cost of electrical energy and concurrent development in the material technology, the operating cost and/or some specific items of performance play a significant role in the overall economics as well as in the efficiency of the system. In this case a design optimization allows to achieve better results.

This leads to the need of an efficient solution to the particular constrained problems which derive from the design optimization of induction motors.

The paper's aim is to show how the results obtained with algorithms, widely used in this field, can be significantly improved by using algorithms which take into consideration all the peculiarities of this class of optimization problems. In particular a new algorithm is presented which is a modified version of the Price method recently proposed in [1].

This algorithm has been used for the design optimization of low voltage three-phase induction motors. The chosen objective functions are either performance oriented or based on some form of cost consideration. They are:

F1) total cost (manufacturing + operating cost);
F2) manufacturing cost;
F3) power factor;
F4) starting torque;
F5) breakdown torque.

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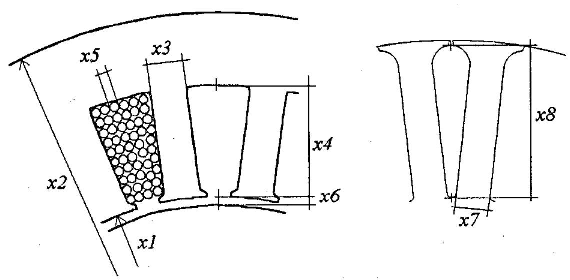

Fig. 1 : Design variables

The independent variables concern the stator and rotor dimensions (Fig.1), the stack length and the number of turns per phase, while mechanical variables such as enclosure, blower and shaft dimension are not considered (because they do not have much influence either on the objective function or on the specified constraints).

The physical description of the motor is reduced to equivalent parameters such as resistances and inductances: the adopted analytical model takes into account the influence of saturation on stator and rotor reactances and the influence of skin effect on rotor parameters. The effects of temperature on motor resistances are computed on the basis of a detailed thermal network. The validity of the model has been verified by means of experimental tests on several three-phase induction motors [2].

The optimization results are satisfactory and point out the goodness of the proposed procedure.

II. Optimization strategies

The design optimization can be formulated as follows:

$$\begin{aligned} &\min f(x) \\ &s.t.\ g(x) \le 0 \\ &l \le x \le u, \end{aligned}$$

where $x \in R^n, f: R^n \to R, g: R^n \to R^p$.

In the sequel the feasible region is denoted as

$$F = \{x \in R^n : g(x) \le 0,\ l \le x \le u\}.$$

The distinguishing features of such an optimization problem are that:

(i) an explicit mathematical representation of the objective function and of constraint functions is not available;

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(ii) the constraints $g_i \leq 0,\ i=1,..p$, are not very restrictive, namely it is relatively easy to find a feasible point and to remain in the feasible region;

(iii) different local minimum points lie beside global minimum points.

The approaches proposed in literature ([3]-[7]) to tackle induction motor design optimization, use one or both the following strategies:

- the constraints $g_i(x) \leq 0,\ i=1,..p$, are eliminated by adding to the objective function an interior penalty term [8] which goes to infinity at the boundary of the set $\{x \in R^n : g_i(x) \leq 0\}$;
- the (possibly modified) objective function is minimized over the box $l \leq x \leq u$ by adopting a derivative-free unconstrained local optimization method; such an algorithm is able to find a stationary point of the objective function, without using first order derivatives (e.g. simplex method [10] or the Hooke-Jeeves algorithm).

Although these approaches have generally produced satisfactory results, they present some drawbacks in obtaining a good approximation of the global minimum point of the original problem.

First of all, let us consider the use of interior penalty functions. This approach is practicable because of feature (ii) belonging to the original problem and consists in transforming a constrained problem into a sequence of unconstrained minimization problems of particular penalty functions. Its interest relies on the fact that each minimization can be performed by means of efficient methods already proposed in literature. Unfortunately these methods need first order derivatives of the objective function, so they cannot be used to minimize the interior penalty function which derives from the motor design because derivatives are not available (see feature (i) of the problem). For this class of minimization problems, the use of an interior penalty function is likely to present another negative aspect. In fact, it can happen that an improvement in the penalty function value does not necessarily imply an improvement in the original objective function (see section IV). Furthermore, the use of penalty methods prevents any algorithm to produce points on the boundary of the set $\{x \in R^n : g_i(x) \leq 0\}$. Consequently it is impossible to locate the global minimum which is often situated on this boundary (see Fig.2 and Fig.3).

With regards to derivative free methods, usually applied to the design optimization of induction motors in the unconstrained minimization of the penalty functions, they are defined to find a stationary point of the objective function. Therefore they are not able to escape from the neighborhood of local minimum points. This implies that:

- rarely they are able to locate the real global minimum point of the original problem;
- the solution point produced by these algorithms strongly depends on the initial point.

In this paper a new algorithm that overcomes the mentioned drawbacks is described. The proposed method does not use any penalty function as in [7] and [9]; instead, because of feature (ii) of the original problem, it is able to produce directly feasible points by dealing differently with the simple box constraints $\{x \in R^n : l \leq x \leq u\}$ and the general nonlinear constraints $\{x \in R^n : g_i(x) \leq 0\}$.

Another interesting feature is that it is a modification of a method which was defined to locate the global minimum of a function and therefore it does not get trapped in local minima.

The algorithm consists in two phases:

I) a global phase performed with random strategy;

II) a local phase that draws its inspiration from the strategy of the simplex method [10] and exploits as much as possible the information on the objective function obtained during the iterations of the algorithm.

III. Modified Price Algorithm

The algorithm can be synthesized as follows:

Data: m such that $m \geq 2n+1$

<u>Step 0</u> (search of an initial set of random points) :
Set k=0; choose at random m vectors $x_i^k\ \ i = 1..m$ over F and define the initial set: $S^k = \{x_1^k ... x_m^k$.

<u>Step 1</u> (search of maximum and minimum function values) :
Determine $x_{\max}^k$, $x_{\min}^k$ and $f_{\max}^k$, $f_{\min}^k$ such that

$$f_{\max}^k = f(x_{\max}^k) = \max_{x \in S^k} f(x) \qquad (1)$$

$$f_{\min}^k = f(x_{\min}^k) = \min_{x \in S^k} f(x) \qquad (2)$$

If $\left| f_{\max}^k - f_{\min}^k \right| \leq 10^{-8}$, then STOP.

<u>Step 2</u> (determination of the weighted centroid) :
Choose at random $n+1$ vectors $x_{i_0}^k$, $x_{i_1}^k$,...., $x_{i_n}^k$ over S^k.
Determine the weighted centroid:

$$c_w^k = \sum_{j=1}^{n} w_j^k x_{i_j}^k \ ; \qquad w_j^k = \frac{\eta_j^k}{\sum_{j=1}^{n} \eta_j^k} \qquad (3)$$

$$\eta_j^k = \frac{1}{f(x_{i_j}^k) - f_{\min}^k + \Phi^k} ; \qquad \Phi^k = 10^3 \frac{\left(f_{\max}^k - f_{\min}^k\right)^2}{f_{\max}^0 - f_{\min}^0}$$

<u>Step 3</u> (determination of the new trial point):
Determine the trial point $\tilde{x}^k$ by performing a weighted reflection. Let:

$$f_w^k = \sum_{j=1}^{n} w_j^k f\left(x_{i_j}^k\right) \qquad (4)$$

then take:

$$\tilde{x}^k = \begin{cases} c_w^k - \alpha^k (x_{i_0}^k - c_w^k) & \text{if } f_w^k \leq f(x_{i_0}^k) \\ x_{i_0}^k - \alpha^k (c_w^k - x_{i_0}^k) & \text{if } f_w^k > f(x_{i_0}^k) \end{cases} \qquad (5)$$

$$\alpha^k = \begin{cases} 1-\dfrac{f(x_{i0}^k)-f_w^k}{f_{\max}^k - f_{\min}^k + \Phi^k} & \text{if } f_w^k \le f(x_{i0}^k) \\ 1-\dfrac{f_w^k - f(x_{i0}^k)}{f_{\max}^k - f_{\min}^k + \Phi^k} & \text{if } f_w^k > f(x_{i0}^k) \end{cases} \quad (6)$$

Step 4 (projection onto box constraints and check of non linear constraints):

For i=*1..n* set $\tilde{x}_i^k = \max\{l_i, \min\{u_i, \tilde{x}_i^k\}\}$ (7)

If $g(\tilde{x}^k) \le 0$ then evaluate $f(\tilde{x}^k)$ else GOTO Step 2.

Step 5 (update of the search set S^k):

if $f(\tilde{x}^k) \ge f_{\max}^k$ then take $S^{k+1} = S^k$

Set k=k+1 and GOTO Step 2.

If $f(\tilde{x}^k) < f_{\max}^k$ then take $S^{k+1} = S^k \cup \{\tilde{x}^k\} - \{x_{\max}^k\}$

Set k=k+1 and GOTO Step 1.

A detailed description and discussion of the motivations of the approach followed by the algorithm is reported in [1]. Here only the main ideas are recalled.

In order to perform a global search on the whole region of interest, at Step 0, m points are randomly generated over F and are stored in the set S^k. At each iteration, the algorithm tries to update the set S^k by substituting the worst point $x_{\max}^k$ contained in S^k with a new point $\tilde{x}^k$ where the objective function value is improved (see Step 5). In this way the points of the set S^k tend to, as the number k of iterations increases, be more and more clustered around the global minimum point of the problem. At Step 1, the goodness of this clustering is checked by comparing the best and the worst objective function values over S^k and when these two values are close, the algorithm stops. At Step 2 and Step 3, the new point $\tilde{x}^k$ is computed. A further explanation of the rationale behind the formulae which produce this point is presented in [1]. Here it is only worth pointing out that these formulae are such that, at initial iterations, $\Phi^k >> f(x_{ij}^k) - f_{\min}^k$ (j = *1..n*) so that $w_j^k \cong \frac{1}{n}$ and $\alpha^k \cong 1$. In this way the point $\tilde{x}^k$ is produced in the same way of the non linear simplex method [10] which does not privilege any particular region of F. As the number of the iterations increases, the scalar Φ^k becomes smaller and smaller, so that w^k and α^k weigh more and more the points where the function values are closer to $f_{\min}^k$.

In this way the new point $\tilde{x}^k$ is produced by exploiting the information on the problem obtained at the preceding iterations of the algorithm so as to privilege the region where more likely a global minimum is located.

IV. Results

The algorithm has been applied to optimize the design of the following three-phase induction motors:

a) 1.5 kW, 220/380 V, 4 pole, 50 Hz;
b) 7.5 kW, 220/380 V, 4 pole, 50 Hz.

The considered objective functions are those already presented in section I: F1, and F2 have been minimized, while F3, F4 and F5 have been maximized. The operating cost in F1 has been calculated with reference to an estimated motor life of 10 years (1840 hours per year). The selected constraints are: stator winding temperature, rotor bars temperature, flux density in the stator and rotor teeth, rated slip, starting current, starting torque (for F1, F2, F3 and F5), breakdown torque (for F1, F2, F3 and F4), power factor at rated load (for F1, F2, F4 and F5) and stator slot fullness.

In order to have an idea of the practical interest of the new approach, the behavior of the new algorithm has been compared with a standard approach one: in particular the algorithm described in [3] and [4] has been considered. In this algorithm, nonlinear constraints are taken into account by adding the term $-\varepsilon\sum_{j=1}^{p} 1/g_j(x)$ (where the positive constant ε is updated during the iterations of the algorithm) to the original objective function. Then the obtained interior penalty function is minimized over the box $\{x \in R^n : l \le x \le u\}$ by using the Hooke-Jeeves method. Each considered problem has been solved ten times by means of the two algorithms.

In the different runs of the Hooke-Jeeves method ten different initial designs have been chosen whose performance are very close to the ones of a commercial motor. In the different runs of the modified Price method ten different initial random sets of m=*250* vectors have been considered.

The average values of the best designs are presented in Table I and Table II: they also show the percentage variations (Δ%) with respect of the cost and performance of commercial motors and the standard deviation (σ) of the ten runs population for the two algorithms. The comparison points out that the new algorithm is more efficient than the algorithm proposed in [3] and produces significant improvements for all the objective functions and both investigated motors. The tests have also pointed out the reliability and robustness of the new method, as can be realized from Fig.2 and Fig. 3 : they show, as an example, the best values of the objective function "Total cost" (F1). In all ten runs, the modified Price algorithm converges to the same optimal point and, hence, gives the same optimal value. On the contrary the Hooke-Jeeves method has an unstable behavior that strongly depends on the starting point. In all runs, it has not been able to locate an optimal point. Furthermore, in the sixth run of Fig.3, the point produced by the Hooke-Jeeves algorithm yields an objective function value which is worse than the starting point one (particularly the initial value was about 2570 US$ while the final about 2580 US$). Finally, Table III and Table IV show the values of nonlinear constraints at the optimal points obtained by modified Price algorithm (e.g. for the objective functions F1, F2 and F3) and the corresponding imposed limit values. These tables point out how the optimal points of this particular class of optimization problems lie on the boundary (see bold values) of the set described by nonlinear constraints. In other words, these points are in a region where every

TABLE I
OPTIMIZATION RESULTS (1.5 KW)

O.F.	Commercial Motor	Modified Price algorithm			Hooke - Jeeves algorithm		
		Average of best values	Δ%	σ	Average of best values	Δ%	σ
F1 (US $)	654	464	-29.1	0	517	-20.9	27.2
F2 (US $)	58	51	-12.1	$2.2\ 10^{-4}$	55	- 5.2	2.70
F3	0.77	0.81	5.2	$1.2\ 10^{-4}$	0.79	2.6	$3.7\ 10^{-3}$
F4 (Nm)	22.7	30.2	33.0	0.35	27.3	20.3	0.89
F5 (Nm)	25.2	31.7	25.8	$4.3\ 10^{-2}$	28.6	13.5	0.89

TABLE II
OPTIMIZATION RESULTS (7.5 KW)

O.F.	Commercial Motor	Modified Price algorithm			Hooke - Jeeves algorithm		
		Average of best values	Δ%	σ	Average of best values	Δ%	σ
F1 (US $)	2500	2050	- 18.0	0	2220	- 11.2	134
F2 (US $)	188	171	- 9.0	$9.1\ 10^{-3}$	175	- 7.0	4.75
F3	0.86	0.90	4.7	0	0.89	3.5	$8.8\ 10^{-3}$
F4 (Nm)	86	146	69.8	0	113	31.4	10.0
F5 (Nm)	146	179	22.6	$6.6\ 10^{-3}$	163	11.6	4.51

TABLE III
1.5 KW - NONLINEAR CONSTRAINTS VALUES AT OPTIMUM (MODIFIED PRICE)

Constraint	Limit value	F1	F2	F3
Stator winding temperature (C)	110	70.8	**110.0**	99.3
Rotor bars temperature (C)	130	79.6	123.8	114.4
Flux density in stator teeth (T)	1.74	1.50	1.55	1.40
Flux density in rotor teeth (T)	1.74	1.58	**1.74**	1.42
Rated slip	0.085	0.047	0.072	**0.085**
Starting current (A)	18.0	17.3	16.3	14.8
Starting torque (Nm)	22.0	**22.0**	22.5	**22.0**
Breakdown torque (Nm)	22.0	29.4	29.9	23.9
Power factor at rated load	0.76	**0.760**	0.763	-
Stator slot fullness	0.50	**0.50**	**0.50**	0.49

TABLE IV
7.5 KW -NONLINEAR CONSTRAINTS VALUES AT OPTIMUM (MODIFIED PRICE)

Constraint	Limit value	F1	F2	F3
Stator winding temperature (C)	135	107.2	**135.0**	132.8
Rotor bars temperature (C)	150	118.1	148.9	146.0
Flux density in stator teeth (T)	1.70	1.46	1.44	1.50
Flux density in rotor teeth (T)	1.80	1.72	1.79	1.35
Rated slip	0.050	0.028	0.038	0.045
Starting current (A)	110	104.8	109.0	93.6
Starting torque (Nm)	80.0	85.9	104.0	87.0
Breakdown torque (Nm)	140	160.3	154.1	**140.0**
Power factor at rated load	0.84	0.87	0.857	-
Stator slot fullness	0.55	**0.55**	**0.55**	**0.55**

interior penalty function is not defined while the proposed approach is able to locate them exactly.

V. CONCLUSIONS

A new algorithm has been presented which is a modified version of the Price method. This algorithm has been used for the design optimization of low voltage three-phase induction motors. The numerical experience seems to point out the limits of an algorithm based on the common approach and the efficiency of an algorithm which directly considers the constraints and uses a strategy defined to locate a global minimum.

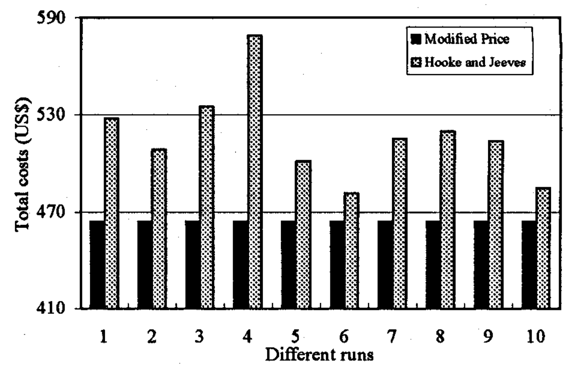

Fig.2 - 1.5 kW: Total cost (O.F. F1) of the optimized designs.

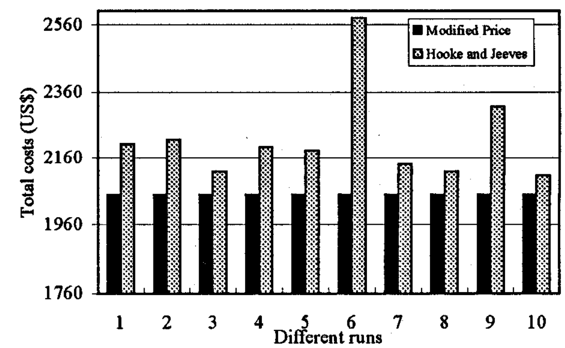

Fig. 3 - 7.5 kW: Total cost (O.F. F1) of the optimized designs.

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