Multiobjective Optimization Techniques for the Design of Induction Motors

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***Abstract*—This paper deals with the optimization problem of induction motor design. In order to tackle all the conflicting goals that define the problem, the use of multiobjective optimization is investigated. The numerical results show that the approach is viable.**

***Index Terms*—Global optimization, induction motor design, multiobjective optimization.**

I. Introduction

THE OPTIMAL design of electrical motors is a difficult problem in that:

1) it involves many variables which nonlinearly affects all the features and the behavior of the electrical apparatus itself;
2) it needs the variables to be chosen in such a way that the design is feasible;
3) it often involves various conflicting objectives and goals.

For these reasons, the designer can profitably state the problem as a nonlinear programming problem able to deal with some or all the aforementioned difficulties and solve it with a suitable numerical optimization technique.

In this paper, the authors model the optimal design of an induction motor as a nonlinear multiobjective optimization problem and describe three different methods for its solution.

When a multiobjective problem is treated, each objective conflicts with one another and, unlike a single objective optimization, the solution to this problem is not a single point, but a family of solutions known as the Pareto-optimal set. Among these solutions, the designer should find the best compromise taking into proper account the attributes and characteristics of the handled problem.

In literature, several multiobjective approaches for the electrical machines design have been proposed [1], [2]; in this paper, three methods are described and employed for the design optimization of three-phase induction motors when conflicting objectives are chosen. An example is presented for a 7.5-kW four-pole motor design, where the objective functions are the manufacturing cost, the rated efficiency, the power factor, and the starting current.

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II. Multiobjective Formulation

The optimal design problem of induction motors can be formulated as the following multiobjective nonlinear optimization problem:

$$\min_{x \in F} f_1(x), f_2(x), \ldots, f_k(x) \quad (1)$$

where $x \in R^n$, $f_i : R^n \rightarrowtail R$, and F is the feasible set of problem (1) which is described by inequalities as follows:

$$F = \{x \in R^n : g_i(x) \leq 0, i = 1, 2, \ldots, p\}. \quad (2)$$

Furthermore, in the considered case, we have some additional features.

1) An analytical representation of $f_i(x)$, $i = 1, 2, \ldots, k$, and $g_i(x)$, $i = 1, 2, \ldots, p$, is not available and, hence, no derivative information can be used in the solution process.
2) The constraints $g_i(x)$, $i = 1, 2, \ldots, p$ are not restrictive in that it is relatively easy to produce a feasible point and remain in the feasible region.

We denote by $f(x) \in R^k$ the vector made up of all the objective functions, that is, $f(x) = (f_1(x), f_2(x), \ldots, f_k(x))$.

An ideal solution of (1) would be a point $x^* \in F$ such that

$$f_i(x^*) \leq f_i(x), \quad \forall x \in F, \quad \forall i \in \{1, \ldots, k\}.$$

Unfortunately, such a point x^* seldom exists, therefore (1) turns into finding some or all the *Pareto optimal* solutions, that is, points satisfying the following definition.

A. Definition

A point $x^* \in F$ is a Pareto-optimal solution of (1) if there does not exist any feasible point $x \in F$ such that

$$f_i(x) \leq f_i(x^*), \qquad \forall i \in \{1, \ldots, k\}$$

and

$$f_j(x) < f_j(x^*)$$

for at least one index $j \in \{1, 2, \ldots, k\}$.

There exists a wide variety of methods that can be used to compute Pareto optimal points.

A widely used technique consists of reducing the multiobjective problem (1) to a single-objective one by means of so-called "scalarization" procedure.

A first choice consists of transforming (1) into a sequence of constrained problems in which a particular objective function

$f_i(x)$ is minimized and the remaining ones are "constrained" to keep their values below a prefixed upper bound.

A second choice consists of minimizing a function which is a "combination" of all the objective functions of (1).

In this paper, we adopt the second strategy since the first one would require an efficient derivative-free algorithm for a global optimization problem with a feasible set complicated by the presence of hard constraints (namely, those involving the bounds on the objective functions).

We employ three different scalarization techniques.

The first one consists of assigning each objective function a cost coefficient and then minimizing the function obtained by summing up all the objective functions scaled by their cost coefficients, that is

$$\min_{x \in F} \sum_{i=1}^{k} c^i f_i(x). \tag{3}$$

The second method consists of choosing an "ideal optimal value" for each objective function and then minimizing the euclidean distance between the actual vector of objective functions and the vector made up of the ideal values, that is

$$\min_{x \in F} \sum_{i=1}^{k} \left(f_i(x) - z_i^{\mathrm{id}}\right)^2 \tag{4}$$

where z_i^{id} stands for the ith ideal value.

The third strategy is similar to the second one in that we consider again the differences $f_i(x) - z_i^{\mathrm{id}}$, $i = 1, 2, \ldots, k$, but this time we consider the following problem:

$$\min_{x \in F} \max_{i=1,2,\ldots,k} \left|f_i(x) - z_i^{\mathrm{id}}\right|. \tag{5}$$

These three methods are well known in literature. The first one is the "cost function method," whereas the other two methods are called "goal programming."

It is possible to prove that each global minimum point of problems (3), (4), and (5) is a Pareto-optimal solution for (1) (see [3]). Obviously, the global solutions of problems (3)–(5) are affected by the coefficients c_i and the ideal values z_i^{id}, $i = 1, 2, \ldots, k$.

Naturally, the choice of these values can be done by exploiting any possible *a priori* knowledge on the underlying optimal design problem. In this paper, we decided to determine these quantities without requiring any *a priori* information. For this reason, we first solve k global optimization problems, namely

$$z_i^* = \min_{x \in F} f_i(x). \tag{6}$$

Then, we set

$$c_i = \frac{1}{z_i^*} \text{ and } z_i^{\mathrm{id}} = z_i^*.$$

III. Global Optimization Algorithm

As described in the previous section, the proposed multiobjective approach for the optimal design of induction motors requires us to find global solutions for (3), (4), (5), or (6). These problems present several local minimum points beside the global ones. Therefore, by taking also into account that analytical representations of the objective functions and constraints are not available, we need a derivative-free global optimization algorithm.

In particular, we have adopted an algorithm recently proposed in [4] which proved to be an efficient tool for the solution of similar global optimization problems. This algorithm, which belongs to the class of Controlled Random Search algorithms, is based on the following points.

1) *Initialization*: The objective function is sampled on a set S of m points randomly chosen within the feasible set F.
2) *Stopping criterion*: If the maximum and minimum value of the objective function over S are sufficiently close to each other, the algorithm stops.
3) *Search phase*: $n + 1$ points are randomly chosen on set S. Then, by exploiting as much as possible the information on the objective function conveyed by these $n + 1$ points, a new point $\tilde{x}$ is produced.
4) *Updating phase*: If the objective function value on the new point is better then the maximum function value, then set S is updated by adding the new point and discarding the worst one. The algorithm continues iterating through the last three steps.

Reference [4] presents a detailed description of this algorithm.

IV. The Objective Functions

The design optimization of electric motors requires a particular attention in the choice of the objective function that usually concerns economic or performance features.

For this reason, we have chosen four conflicting objective functions that can affect the design optimization of three phase induction motors. Particularly:

f_1) Manufacting cost $=$ active material cost $+$ labor cost (to minimize);
f_2) Rated efficiency (to maximize);
f_3) Power factor (to maximize);
f_4) Starting current (to minimize).

However, a good design should represent the right compromise among different objectives but the problem consists in searching this "compromise." The only tool able to solve this problem is represented by the multiobjective approach. Then, we have considered the following multiobjective formulations:

M1)

$$\min_{x \in F} f_1(x), -f_2(x), -f_3(x) \tag{7}$$

M2)

$$\min_{x\in F} f_1(x), -f_2(x), f_4(x) \tag{8}$$

M3)

$$\min_{x\in F} f_1(x), -f_2(x), -f_3(x), f_4(x). \tag{9}$$

This approach allows us to investigate how each single-objective and multiobjective problem affects the results in terms of performance and independent variables and, above all, allows us to have a wide range of alternative designs among which the designer can choose a better solution.

V. Numerical Results

The algorithms have been applied to optimize the design of a typical size in the range $0 \div 22$ kW, that is a 7.5-kW 220/380-V four-pole 50-Hz, single cage induction motor.

The independent variables are related to the stator and rotor dimensions, the stack length, and the stator winding. It is important to underline that the outside stator diameter has not been changed in all optimizations, in order to use the same housing.

Regarding the constraints, they concern mainly the motor performance and we have chosen the following: the stator winding temperature, the rotor bars temperature, the flux density in the stator and rotor teeth, the rated slip, the starting torque, the starting current, (for f_1, f_2, f_3, and M1), the breakdown torque, the power factor at rated load (for f_1, f_2, f_4, and M2), and the stator slot fullness.

Table I shows the optimization results with reference to the single objectives [Manufacturing cost (f_1), efficiency (f_2), power factor (f_3), starting current (f_4)] and the multiobjective formulations (M1, M2, and M3). The last ones have been solved by means of the scalarization methods (3)–(5).

As regards the choice of the ideal values in the multiobjective methods, we decided to impose the best values obtained by the single-objective optimizations $f_{1,}f_{2,}, f_3$, and f_4 (bold values in Table I).

Table II presents, for each optimization, the values assumed by the independent variables.

We remark that each row in Table I represents a different Pareto-optimal solution of (1) and, as such, there does not exist any one better then all the others. In other words, there is no way to compare them all. In fact, given two Pareto points, one is better then the other with respect to at least one objective, and vice versa.

All we propose, in order to facilitate the decision maker or the designer in choosing the right design, is to graph these solutions cleverly.

To obtain sensible graphs, we scale the four objectives, f_1, f_2, f_3, and f_4 so as to obtain maximum values equal to one.

In Fig. 1, one piecewise linear curve for every row of Table I is shown, that is for each Pareto optimal solution. Each curve connects points corresponding to the scaled values of the four considered objectives.

TABLE I
Optimization Results

	Manufacturing Cost (f_1)	Efficiency (f_2)	Power Factor (f_3)	Start. Current (f_4)
min (f_1)	**148.1 €**	0.863	0.804	55.6 A
min ($-f_2$)	190.3 €	**0.911**	0.804	58.9 A
min ($-f_3$)	167.8 €	0.876	**0.880**	44.5 A
min (f_4)	180.6 €	0.887	0.861	**41.7 A**
M1 (3)	151.4 €	0.875	0.869	45.4 A
M1 (4)	151.6 €	0.877	0.867	46.3 A
M1 (5)	152.5 €	0.884	0.854	49.6 A
M2 (3)	151.3 €	0.873	0.864	44.4 A
M2 (4)	154.2 €	0.876	0.861	43.8 A
M2 (5)	155.2 €	0.876	0.861	43.7 A
M3 (3)	150.9 €	0.872	0.865	44.5 A
M3 (4)	153.4 €	0.875	0.862	44.0 A
M3 (5)	155.1 €	0.876	0.861	43.7 A

TABLE II
Independent Variables

	L (mm)	D_I (mm)	h_{SS} (mm)	w_{ST} (mm)	h_{RS} (mm)	w_{RT} (mm)	S_{CU} (mm^2)	N_{CS}
min (f_1)	140	125.5	16.4	6.5	16.0	5.0	1.40	32
min ($-f_2$)	190	122.0	19.0	4.5	18.5	3.5	2.40	29
min ($-f_3$)	152	130.0	17.2	4.8	16.0	4.5	1.76	37
min (f_4)	190	122.0	19.0	5.4	18.5	5.0	2.10	30
M1 (3)	140	128.5	16.9	6.5	18.5	4.8	1.40	35
M1 (4)	140	128.0	16.7	6.3	18.5	4.7	1.40	35
M1 (5)	140	130.0	16.1	6.4	18.5	4.4	1.40	34
M2 (3)	140	125.5	17.4	6.5	18.5	4.9	1.44	34
M2 (4)	143	124.0	18.6	6.5	18.5	4.8	1.54	34
M2 (5)	145	123.5	18.9	6.5	18.5	4.8	1.56	34
M3 (3)	140	126.0	17.2	6.5	18.5	5.0	1.42	34
M3 (4)	140	123.5	19.0	6.5	18.5	4.7	1.53	35
M3 (5)	144	123.0	19.0	6.5	18.5	4.8	1.56	34

($L =$ Stack Length; $D_I =$ Inner Stator Diameter; $h_{SS} =$ Stator Slot Heigh; $w_{ST} =$ Stator Tooth Width; $h_{RS} =$ Rotor Slot Heigh; $w_{RT} =$ Rotor Tooth Width; $S_{CU} =$ Stator Wire Size; $N_{CS} =$ Conductors per Slot).

Obviously, there does not exist a curve which passes below all the others. In fact, if such a curve existed, the represented Pareto solution would be better than any other with respect to all the objectives.

In Fig. 2, we have, again, one stacked bar for every row of Table I. Each bar consists of four sections, one for each objective, f_1, f_2, f_3, and f_4. In this case, we were also able to order the Pareto solutions simply ordering the bars by their heights, that is, by the sum of the four scaled objective values.

The optimization results suggest the following comments.

1) The minimization of the single objective function, gives rise to a significant improvement of the chosen term, but affects heavily the other ones. Particularly, the optimization of motor efficiency (f_2) allows to reach a very interesting result (0.911) that permits to label the motor as "high efficiency motor" according to the new European classification scheme [5]. This goal has required the highest manufacturing cost and starting current, with respect to the other solutions, and a power factor close to the minimum imposed value. The values assumed by the independent variables

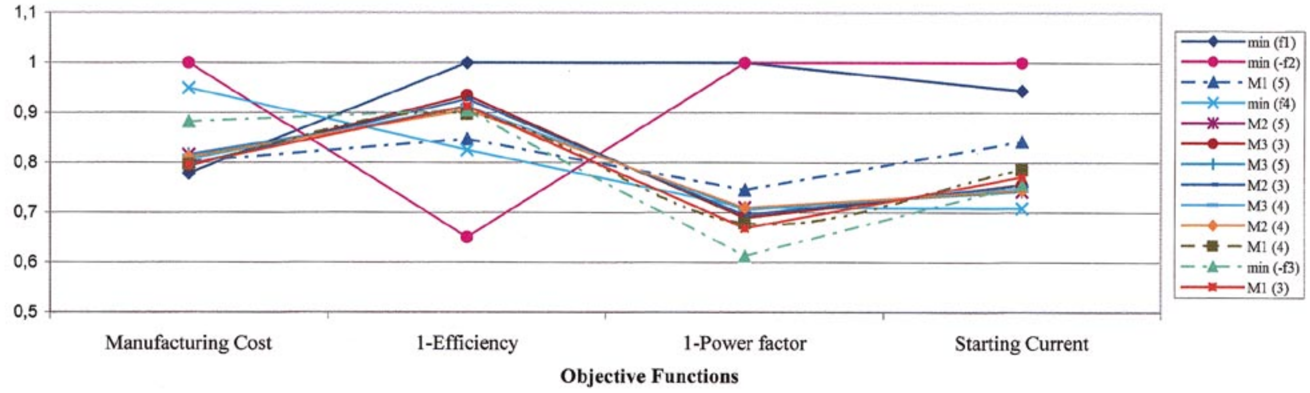

Fig. 1. The objective functions values for different optimizations.

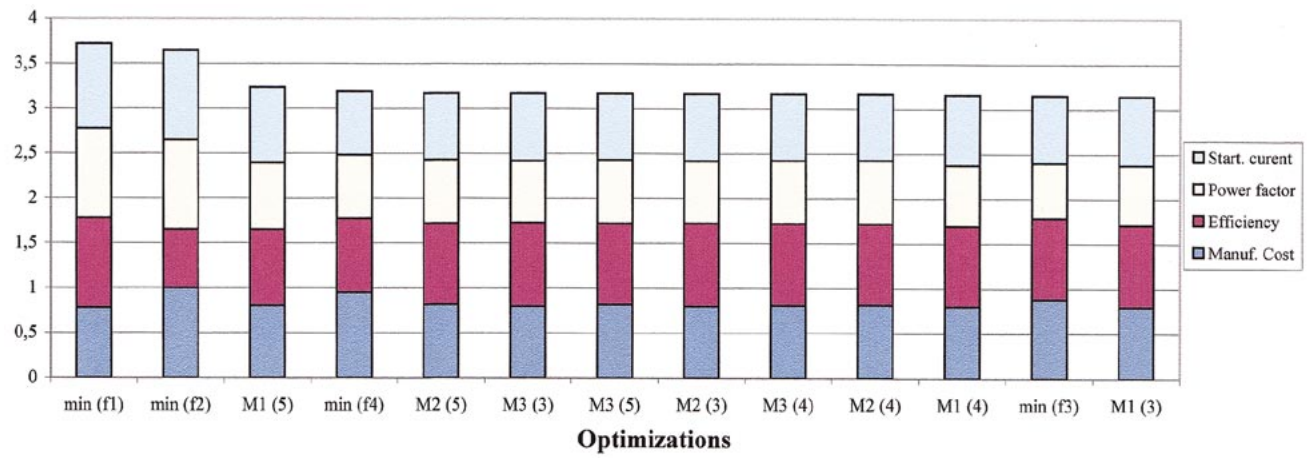

Fig. 2. Sum of the four scaled objective values for different optimizations.

(Table II) point out an increase of stack length (L) and stator wire size (S_{CU}).

2) The optimization of power factor (f_3) gives rise to a satisfactory efficiency and lower starting current.

3) When the manufacturing cost is selected as objective function (f_1), the active material weight reduces and this affects obviously the efficiency and power factor.

4) The multiobjective solutions represent a good compromise and this is well evident by comparing the results obtained with M1–M3 with the minimization of the single objective functions. The manufacturing costs does not exceed 156 E which is lower than the values obtained with the single objective functions f_2, f_3, and f_4. The efficiencies vary in a small range (around 0.875) with the exception of the solution "M1 (5)" that, unfortunately, presents an higher starting current.

The presented results and the behaviors shown in Figs. 1 and 2 point out, for the multiobjective approach, how the designs are quite similar and this confirms the good agreement of the proposed method. However, if we would like "to extract" the better design, the choice should be oriented toward the "M1 (3)" design that represents a better compromise.

VI. Conclusion

The design optimization of the induction motor as a nonlinear multiobjective optimization problem has been investigated and three different methods for its solution have been described.

A 7.5-kW four-pole motor has been optimized by choosing four conflicting objective functions.

The obtained results are satisfactory and point out the effectiveness of a multiobjective approach since it allows us to find a good compromise among the proposed goals, and above all it represents an efficacious tool for the designer.

References

[1] W. Jazdzynski, "Multicriterial optimization of squirrel-cage induction motor design," *Proc. Inst. Elect. Eng.*, vol. 136, no. 6, pp. 299–307, 1989.

[2] B. Mirzaeian, M. Moallen, V. Tahani, and C. Lucas, "A new multi-objective optimization method based on genetic algorithm and its application for optimal design of a switched reluctance motor," *Proc. ICEM 2000*, vol. 3, pp. 1675–1678.

[3] K. Miettinen, *Nonlinear Multiobjective Optimization*. New York: Kluwer, 1999.

[4] L. Cirio, S. Lucidi, F. Parasiliti, and M. Villani, "A global optimization approach for the synchronous motors design by finite element analysis," Dip. di Informatica e Sistemistica "A. Ruberti," Univ. di Roma "La Sapienza".

[5] F. Parasiliti and M. Villani, *Energy Efficiency Improvements in Electric Motors and Drive*. New York: Springer, 1999.